LESSON 4.3a

Polynomial Long Division

Today you will:

- Divide polynomials using long division
- Practice using English to describe math processes and equations

Core Vocabulary:

- polynomial long division, p. 174

Previous:

$$\boldsymbol{dividend \div divisor = quotient + \frac{remainder}{divisor}}$$

- long division
- dividend - the number being divided … the number on top

$$\boldsymbol{divisor\overline{\big)\, dividend}}^{\;\boldsymbol{quotient}}$$

- divisor - the “divide by” number … the number on the bottom
- quotient - the result of the division, the answer
- remainder - what is left over when the divisor does not evenly go into the dividend

$$\boldsymbol{\frac{dividend}{divisor} = quotient + \frac{remainder}{divisor}}$$

Use long division to solve $6543 \div 12$

$$\begin{array}{r} 545 \\ 12\,\overline{)\,6543} \\ 60 \\ \hline 54 \\ 48 \\ \hline 63 \\ 60 \\ \hline 3 \end{array}$$

- 545 ⟵ quotient
- Multiply divisor by $\frac{65}{12} = 5.41$ so first multiply by 5
- Subtract. Bring down next term
- Multiply divisor by $\frac{54}{12} = 4.5$ so multiply by 4
- Subtract. Bring down next term
- Multiply divisor by $\frac{63}{12} = 5.25$ so multiply by 5
- 3 ⟵ remainder

Answer: 545 remainder 3

or

$$545 + \frac{3}{12}$$

Long Division of Polynomials

When you divide a polynomial $f(x)$ by a nonzero polynomial divisor $d(x)$, you get a quotient polynomial $q(x)$ and a remainder polynomial $r(x)$.

…when you divide polynomials, the answer including the remainder is a polynomial…

$$\frac{f(x)}{d(x)} = q(x) + \frac{r(x)}{d(x)}$$

Divide $2x^4 + 3x^3 + 5x - 1$ by $x^2 + 3x + 2$.

SOLUTION

Write polynomial division in the same format you use when dividing numbers. Include a "0" as the coefficient of x^2 in the dividend. At each stage, divide the term with the highest power in what is left of the dividend by the first term of the divisor. This gives the next term of the quotient.

$$\begin{array}{r l}
 & 2x^2 - 3x + 5 \quad \leftarrow \text{quotient} \\
x^2 + 3x + 2 \overline{)} & 2x^4 + 3x^3 + 0x^2 + 5x - 1 \\
 & \underline{2x^4 + 6x^3 + 4x^2} \\
 & -3x^3 - 4x^2 + 5x \\
 & \underline{-3x^3 - 9x^2 - 6x} \\
 & 5x^2 + 11x - 1 \\
 & \underline{5x^2 + 15x + 10} \\
 & -4x - 11 \quad \leftarrow \text{remainder}
\end{array}$$

Multiply divisor by $\frac{2x^4}{x^2} = 2x^2$.

Subtract. Bring down next term.

Multiply divisor by $\frac{-3x^3}{x^2} = -3x$.

Subtract. Bring down next term.

Multiply divisor by $\frac{5x^2}{x^2} = 5$.

COMMON ERROR

The expression added to the quotient in the result of a long division problem is $\frac{r(x)}{d(x)}$, not $r(x)$.

$$\frac{2x^4 + 3x^3 + 5x - 1}{x^2 + 3x + 2} = 2x^2 - 3x + 5 + \frac{-4x - 11}{x^2 + 3x + 2}$$

$$\frac{2x^4 + 3x^3 + 5x - 1}{x^2 + 3x + 2} = 2x^2 - 3x + 5 + \frac{-4x - 11}{x^2 + 3x + 2}$$

Check

You can check the result of a division problem by multiplying the quotient by the divisor and adding the remainder. The result should be the dividend.

$(2x^2 - 3x + 5)(x^2 + 3x + 2) + (-4x - 11)$

$= (2x^2)(x^2 + 3x + 2) - (3x)(x^2 + 3x + 2) + (5)(x^2 + 3x + 2) - 4x - 11$

$= 2x^4 + 6x^3 + 4x^2 - 3x^3 - 9x^2 - 6x + 5x^2 + 15x + 10 - 4x - 11$

$= 2x^4 + 3x^3 + 5x - 1$ ✓

Divide using long division:

$$(x^3 - x^2 - 2x + 8) \div (x - 1)$$

$$\begin{array}{r|l} & \quad\quad\;\; x^2 - 2 \\ \hline x - 1 & x^3 - x^2 - 2x + 8 \\ & -(x^3 - x^2) \\ \hline & \quad\quad\;\; 0 \;\; - 2x + 8 \\ & \quad\quad\; -(-2x + 2) \\ \hline & \quad\quad\quad\quad\quad\;\; 6 \end{array}$$

$$Answer: x^2 - 2 + \frac{6}{x - 1}$$

Divide using long division:

$$(x^4 + 2x^2 - x + 5) \div (x^2 - x + 1)$$

$$\begin{array}{r} x^2 + x + 2 \\ x^2 - x + 1 \overline{\smash{\big)}\, x^4 + 0x^3 + 2x^2 - x + 5} \\ -(x^4 - x^3 + x^2) \\ \hline x^3 + x^2 - x \\ -(x^3 - x^2 + x) \\ \hline 2x^2 - 2x + 5 \\ -(2x^2 - 2x + 2) \\ \hline 3 \end{array}$$

$$Answer: x^2 + x + 2 + \frac{3}{x^2 - x + 1}$$

Homework

Pg 177, #5-18, 37 (using long division for all problems)